AD-A039 559 CONTROL DATA CORP MELVILLE N Y TRG DIV F/G 17/1

NON-LINEAR PROCESSING OF FAR FIELD NOISE (OR REVERBERATION) IN --ETC(U)

SEP 65 H STEINBERG NOBSR-93023

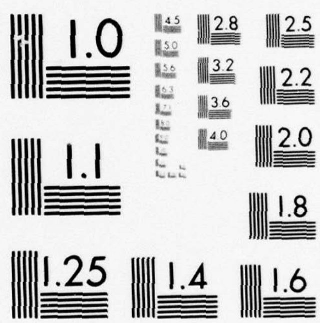
UNCLASSIFIED TRG-023-TN-65-10 NL

| OF |

AD A039559

END

DATE FILMED 6-77



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

MOST Project -5

AD A039559

6

NON-LINEAR PROCESSING OF FAR FIELD NOISE (OR REVERBERATION) IN A SONAR ARRAY SYSTEM,

1

B.S.

by

10 H./Steinberg

12 28 p.

14 TRG Report No. 023-TN-65-10

Contract NObsr-93023 13

D D C
RECEIVED
MAR 10 1977
C

Submitted to:

Navy Electronics Laboratory
San Diego, California

Approved:	Approved:
Walton Graham	
Walton Graham Department Head, TRG	Marvin Baldwin Project Technical Director, NEL

~~TRG~~ Incorporated DIV
A Subsidiary of
Control Data Corporation
Route 110
Melville, New York

11 Sep[redacted]965

AD No. ____
DDC FILE COPY

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

.353 415

GP.1

TABLE OF CONTENTS

ACCESSION for
NTIS White Section ☑
DDC Buff Section ☐
UNANNOUNCED ☐
JUSTIFICATION Per Ltr. on file
BY
DISTRIBUTION/AVAILABILITY CODES
Dist. AVAIL. and/or SPECIAL
A

ABSTRACT

The effect of an anisotropic reverberation pattern on the performance of various signal processing systems for linear sonar arrays was studied. It was determined that the use of clipping at each array element could seriously affect the performance of the system in comparison to a similar analog system. A comparison between square law and multiplier-correlator detection schemes indicated no significant performance difference.

HISTORICAL BACKGROUND

In order to utilize the planar array sonar system to its fullest capacity it is necessary to determine the "best" choice of signal processing and beam forming circuitry. This decision will be ultimately based on a combination of cost, feasibility, and performance considerations. To this end we have analyzed the performance of several systems which can meet the engineering feasibility and cost requirements.

The studies involved in this problem involve two areas of research. First, what sort of environment should we consider, i.e., what is the nature of the signal and noise that will be encountered in practice? Second, how effective is the circuitry in extracting the signal from the noise? Before proceeding, let us give some historical background in these areas.

In the early history of noise analysis (e.g., Rice (1)), random noise was considered to be a stationary Gaussian process. As long as one considered only single receiver systems, this model proved to be adequate for practical analysis. Later a theory of multiple receiver systems was developed (e.g. Faran & Hills (2) and (3)), and it became necessary to develop a model to include noise cross-correlation effects, which required some picture of the distribution in space of noise sources. To obtain this cross-correlation, Jacobsen (4) examined homogeneously distributed sources which led to an isotropic distribution of noise at the receiver system. In many realistic situations the isotropic assumption breaks down and non-isotropic fields must be considered (Bryn (5)). Furthermore, it has been found in practice that certain non-Gaussian effects (sometimes erroneously called non-stationary) must be considered. These effects include random amplitude modulation (Thomas and Williams (6)) and random phase fluctuations (Berman and Berman (7)).

Paralleling this development is the evolution of noise models based on a more physical approach to the problems under consideration. In sonar work these problems appear to fall into three main categories: impulse noise, covering a wide range of phenomena, such as "static" in the electronics, wave slaps, shrimp in Chesapeake Bay, etc., flow noise, and reverberation. Impulse noise has been studied by Middleton (8) and others, principally in relation to radar and communications, rather than sonar. Flow noise, important at high speeds, is related to pressure fluctuations of the boundary layer, which has been studied by Kraichnan (9), Corcos (10), Gardner (11), and others, and to cavitation at the boundary. Reverberation has been examined by many authors (e.g., Stewart (12), Westerfield (13), Faure (14)), and seems to be the principal problem at low ship speeds.

When we examine the circuitry we can for the purposes of analysis immediately divide the elements into two categories, linear and non-linear. Linear problems involve determining the optimum frequency response of filters (Childers & Reed (15)), the number of elements, geometry, and amplitude and phase responses of receivers (Bryn (5)), methods of digital sampling (Rudnick (16)), etc. Non-linear problems concern the use of clipping (Gore (17), Rudnick (16)), and detection instrumentation, e.g., choice between square law or some sort of mulitplier correlation scheme (Faran & Hills (3)).

In this paper we have studied the non-linear processing with the particular emphasis on realistic noise models. The problems of particular interest are the effect of clipping when the noise background is non-isotropic, and also a comparison under these conditions of various combinations of clipping (non-clipping) and detection schemes.

SECTION I

INTRODUCTION

In this note we are concerned with the problem of determining as precisely as possible the angular distribution of noise and signal with the objective of determining if signals are present and if so in what direction.

Once the filter of the detection system is matched to that of the anticipated signal, the only characteristic which distinguishes signal from noise is the fact that the signal is concentrated in one direction, so that it would appear as a local spike in the angular pattern of signal plus noise. Furthermore, if (as in the case of an active pulse system) range information is also available, the signal can be seen as a spike in the range distribution in a given direction.

For this problem, a linear array of hydrophones is used as the reception system, and we are concerned with ascertaining the efficiency of various processing systems. In particular we are concerned with (1) the effect of clipping and (2) a comparison of square law with multiplier-correlator detection, for an arbitrary (anisotropic) noise background.

In order to compare the various systems, we have adopted as a figure of merit the signal/noise ratio (as defined by Faran & Hills (2)) i.e., the ratio of the difference of the on-target and off-target mean outputs to the r.m.s. value of the on-target output. Although, in general, this criterion can be very misleading in many situations, e.g., if the smoothing times were different, for these systems (particularly for arrays of many elements) we can use the results for qualitative comparisons.

SECTION II

SYSTEMS DESCRIPTIONS

The basic hardware in the system under study is a linear array of M (0,..., M-1) hydrophone receivers, each followed by a band pass filter, with uniform spacing between adjacent elements. In order to determine the direction of a signal, each array element uses an associated delay device, and in all cases analyzed we will assume a uniform spacing of the delay times, i.e., the $n^{\underline{th}}$ element ($n = 0,\ldots, M-1$) will have its input delayed $n\Delta$, where Δ is some time interval.

Given this basic setup, we are concerned with comparing four different signal processing systems, as follows:

1. The output (i.e., the delayed signals) are added, then the resultant is squared and finally smoothed by a simple time averaging.

2. The array is divided into an upper and a lower half, (elements (0, ..., L-1), (L, ... , M-1)). The outputs of each half are summed, then the two sums are multiplied together and the resultant is smoothed.

3. Same as 1., except that before adding, each output is infinitely clipped, i.e., only the polarity information is retained at each hydrophone. (Clipping and delay commute, so that the question of which to do first is determined purely by hardware considerations.) Furthermore, since clipping generates higher harmonics, it is necessary to filter the clipped signal before squaring. (See Gore (17)).

4. Same as 3., except that after the second filtering, the array is divided into two halves and the final processing is the same as in 2.

SECTION III

NOISE DESCRIPTION

For this problem we assume that the noise sources are at distances sufficiently large compared to the dimensions of the array, so that the sound waves from any direction are parallel as they encounter the array. However, we do not assume that the noise is isotropic at any given time. Although it is not necessary, we assume the array is backed by an infinite baffle so that only sources in a hemisphere are received.

Let θ be the angle between the direction of a noise source and the array, and let ϕ denote the angle with a vertical axis (the array is assumed horizontal), then $u(\theta, \phi, t)$ will denote the pressure at time t at the $0^{\underline{th}}$ element of the array arising from noise in the (θ,ϕ) direction. Let d denote the distance between adjacent array elements and c the speed of sound, then the pressure at the $n^{\underline{th}}$ element is then $u(\theta, \phi, t + \frac{nd}{c} \cos \theta)$.

Let $a_n(t)$ be the total sound pressure at time t at the $n^{\underline{th}}$ element, then

$$a_n(t) = \frac{1}{2\pi} \int_0^{\pi} \int_0^{\pi} u(\theta,\phi,t + \frac{nd}{c} \cos \theta) \sin \theta \, d\theta \, d\phi$$

To complete the model we will assume

$$E(u(\theta,\phi,t)u(\theta',\phi',t')) = 2\pi P(\theta,\phi)\delta(\cos\theta-\cos\theta')\delta(\phi-\phi')\rho(t-t')$$

where $P(\theta,\phi)$ is the noise power in the (θ,ϕ) direction and $\rho(s)$ is the noise autocorrelation function (which we are assuming independent of direction since it will be determined by receiver filter characteristics). The anisotropy in the noise power can be considered to be a product of anisotropy in source and in transmission.

When a signal is present, it is considered as noise, which is distinguishable from the background as a delta function in some direction.

SECTION IV

GENERAL DERIVATION

Let $a_n(t)$ be the amplitude of the pressure at the $n^{\underline{th}}$ receiver at time t, let G_k be the output of the $k^{\underline{th}}$ system, then we have (where Δ is the relative delay between adjacent receivers) k=1, no clipping-square law

$$G_1 = \frac{1}{T} \int_o^T \left(\sum_o^{M-1} a_n(t-n\Delta) \right)^2 dt$$

k=2, no clipping - multiplier

$$G_2 = \frac{1}{T} \int_o^T \left(\sum_o^{L-1} a_n(t-n\Delta) \right) \left(\sum_L^{M-1} a_n(t-n\Delta) \right) dt$$

Let

$$p(t,z(t)) = \int_o^t f(t-\tau) z(\tau) d\tau$$

be the output of a filter, which has an impulse response f(t), with input z(t). Then for

k = 3, clipping - square law

$$G_3 = \frac{1}{T} \int_o^T \left[p\left(t, \sum_o^{M-1} \text{sgn} \left[a_n(t-n\Delta) \right] \right) \right]^2 dt$$

k=4, clipping - multiplier

$$G_4 = \frac{1}{T} \int_o^T p\left(t, \sum_o^{L-1} \text{sgn}\left[a_n(t-n\Delta) \right] \right) p\left(t, \sum_L^{M-1} \text{sgn}\left[a_n(t-n\Delta) \right] \right) dt$$

Let us assume that there is a signal in the direction Θ_o, then if $\Delta_o = \frac{d}{c} \cos \Theta_o$, the mean on-target response of the $k^{\underline{th}}$ system is $E(G_k(\Delta_o))$. To obtain the off-target response there are two alternative approaches, (1) choose a range of angles near Θ_o, sufficiently far away so as not to be in the main beam steered to Θ_o, and take some weighted average of the mean responses at these angles, or (2) assume the signal is absent and calculate the resultant $E(G_k(\Delta_o))$.

Although the former is a more realistic approach for most practical situations, the latter has the advantage of computational ease, and further the difference between the results of the two procedures would in general be negligible. Therefore our figure of merit R_k is given by

$$R_k = \frac{E(G_k(\Delta_o)) - E(G_k^*(\Delta_o))}{\sigma(G_k(\Delta_o))}$$

where G_k^* indicates the output when the signal is absent. In passing, note that Faran and Hills used as their criterion R_k^2. The immediate problem then is to determine $E(G_k)$ and $\sigma(G_k)$ as functions of Δ. Let us introduce some additional notations:

$$\text{Let} \quad \nu_{n_1,n_2}(t_1,t_2) = E(a_{n_1}(t_1)a_{n_2}(t_2))$$

$$\tilde{\nu}_{n_1,n_2}(t_1,t_2) = E\left(\text{sgn}\left[a_{n_1}(t_1)a_{n_2}(t_2)\right]\right)$$

Since we are dealing with Gaussian variables, we have (see Appendix B)

$$\tilde{\nu}_{n_1,n_2}(t_1,t_2) = \frac{2}{\pi} \sin^{-1}\left(\frac{\nu_{n_1,n_2}(t_1,t_2)}{Z}\right)$$

where $Z = E(a_n(t)^2)$, independent of n and t, is the mean power at any element. To calculate $\sigma(G_k)$, we need the fourth order moments

$$E\left(a_{n_1}(t_1)a_{n_2}(t_2)a_{n_3}(t_3)a_{n_4}(t_4)\right) = \nu_{n_1,n_2}(t_1,t_2)\ \nu_{n_3,n_4}(t_3,t_4)$$

$$+ \nu_{n_1,n_3}(t_1,t_3)\ \nu_{n_2,n_4}(t_2,t_4) + \nu_{n_1,n_4}(t_1,t_4)\ \nu_{n_2,n_3}(t_2,t_3),$$

(since the process is Gaussian). For the clipped case in general $E\left(\text{sgn}\left[\prod_{k=1}^{4} a_{n_k}(t_k)\right]\right)$ cannot be represented exactly (see Placket (18)). However, in the large array case since we are concerned only with the products of sums of the form $\sum_{n_k} a_{n_k}(t_k)$, where the number of terms is large, we may assume an approximate Gaussian behavior (see Tze-Chien Sun (19)), at least to the extent that the fourth order moments may be expressed in terms of the second moments, i.e.,

Let $$\Sigma_k = \sum_{n_k} \operatorname{sgn}\left(a_{n_k}\left(t_{n_k}\right)\right)$$

Then

$$E\left(\prod_{k=1}^{4} \Sigma_k\right) \approx E(\Sigma_1\Sigma_2)\ E(\Sigma_3\Sigma_4) + E(\Sigma_1\Sigma_3)\ E(\Sigma_2\Sigma_4) + E(\Sigma_1\Sigma_4)\ E(\Sigma_2\Sigma_3)$$

and

$$E(\Sigma_k\Sigma_j) = \frac{2}{\pi} \sum_{n_k} \sum_{n_j} \sin^{-1}\left(\frac{\nu_{n_k,n_j}\left(t_{n_k}, t_{n_j}\right)}{Z}\right)$$

Then for k = 1,2

$$E(G_k) = \frac{1}{T} \int_o^T \sum_o^{I_k-1} \sum_{J_k}^{M-1} \nu_{n,m}(t-n\Delta,\ t-m\Delta)\,dt$$

$$\sigma^2(G_k) = \frac{1}{T^2} \int_o^T \int_o^T \sum_o^{I_k-1} \sum_{J_k}^{M-1} \sum_o^{I_k-1} \sum_{J_k}^{M-1}$$

$$\Big[\nu_{n_1,n_3}(t_1-n_1\Delta, t_2-n_3\Delta)\ \nu_{n_2,n_4}(t_1-n_2\Delta, t_2-n_4\Delta)$$

$$+\nu_{n_1,n_4}(t_1-n_1\Delta,\ t_2-n_4\Delta)\ \nu_{n_2,n_3}(t_1-n_2\Delta, t_2-n_3\Delta)\Big]\,dt_1\ dt_2$$

where $I_1 = M,\ I_2 = L$
$J_k = M-I_k$

For k = 3,4

$$E(G_k) = \frac{2}{\pi T} \int_o^T \int_o^t \int_o^t f(t-\tau_1)\ f(t-\tau_2)$$

$$\sum_o^{I_k} \sum_{J_k}^{M-1} \sin^{-1}\left(\frac{\nu_{n,m}(\tau_1-n\Delta, \tau_2-m\Delta)}{Z}\right) d\tau_1\ d\tau_2\ dt$$

$$\sigma^2(G_k) = \left(\frac{2}{\pi T}\right)^2 \int_o^T \int_o^T \int_o^{t_2} \int_o^{t_2} \int_o^{t_1} \int_o^{t_1} f(t_1-\tau_1) f(t_1-\tau_2) f(t_2-\tau_3) f(t_2-\tau_4)$$

$$\sum_{o}^{I_k-1}\sum_{J_k}^{M-1}\sum_{o}^{I_k-1}\sum_{J_k}^{M-1}\Bigg[\sin^{-1}\left(\frac{\nu_{n_1,n_3}\left(\tau_1-n_1\Delta,\tau_3-n_3\Delta\right)}{Z}\right)\sin^{-1}\left(\frac{\nu_{n_2,n_4}\left(\tau_2-n_2\Delta,\tau_4-n_4\Delta\right)}{Z}\right)$$

$$+\sin^{-1}\left(\frac{\nu_{n_1,n_4}\left(\tau_1-n_1\Delta,\tau_4-n_4\Delta\right)}{Z}\right)\sin^{-1}\left(\frac{\nu_{n_2,n_3}\left(\tau_2-n_2\Delta,\tau_3-n_3\Delta\right)}{Z}\right)\Bigg]$$

$$d\tau_1 d\tau_2 d\tau_3 d\tau_4 dt_1 dt_2$$

where $I_1=I_3$, $I_2=I_4$, and $J_k=M-I_k$ (as above). Using the representation given in Section III

$$\nu_{n,m}(t_1,t_2) = \frac{1}{2\pi}\int_0^{\pi}\int_0^{\pi} P(\theta,\emptyset)\rho\left(t_1-t_2+\frac{(n-m)d}{c}\cos\theta\right)\sin\theta\, d\theta\, d\emptyset$$

Let us assume that the received spectrum is given by $F(\omega)$, i.e.,

$$\rho(t) = \int_0^{\infty} F(\omega)\cos 2\pi\,\omega t\, d\omega$$

Let

$$D_{I,J}(x_o,t) = \frac{1}{2}\int_{-1}^{1} Q(x)\int_0^{\infty} F(\omega)\, H_{I,J}\left(\omega(x-x_o),\omega t\right) d\omega dx$$

$$E_{I,J}(x_o,t) = \frac{2}{\pi}\sum_{j=o}^{\infty}\frac{\lambda_j^2}{j+1}\int_0^{\infty}\int_0^{\infty}\int_{-1}^{1}\int_{-1}^{1} H_{I,J}\left(\eta_j,\Omega_j t\right)\prod_{\ell=1}^{2j+1} F(\omega_\ell)\frac{Q(x_\ell)}{2Z}dx_\ell\, d\omega_\ell$$

where

$$H_{I,J}(u,v) = \sum_{n=o}^{I-1}\sum_{m=o}^{I-1}\cos\left(2\pi\left(v+\frac{d}{c}(n-m+J)u\right)\right)$$

$$x = \cos\theta$$

$$x_o = \cos\theta_o$$

$$\Delta_o = \frac{d}{c}x_o$$

$$Q(x) = \frac{1}{\pi} \int_0^{\pi} P(\theta,\phi)\,d\phi$$

$$\lambda_j = \frac{(2j)!}{2^{2j}(j!)^2}$$

$$\eta_j = \left(\sum_{\ell=1}^{j} - \sum_{\ell=j+1}^{2j+1} \right) \omega_\ell (x_\ell - x_o)$$

$$\Omega_j = \left(\sum_{\ell=1}^{j} - \sum_{\ell=j+1}^{2j+1} \right) \omega_\ell$$

Then for k = 1 or 2

$$E(G_k) = D_{I_k,J_k}(x_o,0)$$

$$\sigma^2(G_k) = \frac{2}{T^2} \int_0^T (T-t) \left(D^2_{I_k,0}(x_o,t) + D_{I_k,J_k}(x_o,t)\; D_{I_k,J_k}(x_o,-t) \right) dt$$

and for k = 3,4

$$E(G_k) = E_{I_k,J_k}(x_o,0)$$

$$\sigma^2(G_k) = \frac{2}{T^2} \int_0^T (T-t) \left(E^2_{I_k,0}(x_o,t) + E_{I_k,J_k}(x_o,t) E_{I_k,J_k}(x_o,-t) \right) dt$$

(after expanding $\sin^{-1}$ in a power series and using the results in Appendix C).

We can simplify the expression for $E_{I,J}(x_o,t)$ as follows: Let

$$g_n(x_o,t) = \int_0^{\infty} \int_{-1}^{1} e^{-2\pi i\omega \left(n\frac{d}{c}(x-x_o)+t \right)} F(\omega)\,\frac{Q(x)}{2Z}\,dx\,d\omega$$

Then

$$E_{I,J}(x_o,t) = \frac{2}{\pi}\mathrm{Re} \left\{ \sum_{n=0}^{I-1} \sum_{m=0}^{I-1} \sum_{j=0}^{\infty} \frac{\lambda_j^2}{j+1} \left| g_{n-m+J}(x_o,t) \right|^{2j} g_{n-m+J}(x_o,t) \right\}$$

$$= \frac{8}{\pi^2} \sum_{n=0}^{I-1} \sum_{m=0}^{I-1} B\left(\left| g_{n-m+J}(x_o,t) \right| \right) \mathrm{Re}\left(g_{n-m+J}(x_o,t) \right)$$

When B is a complete Elliptic integral (see Jahnke and Emde (20) p. 73). Therefore

$$E_{I,J}(x_o,t) = \frac{4}{Z\pi^2} \int_{-1}^{1} Q(x) \int_{o}^{\infty} F(\omega) \sum_{n=0}^{I-1} \sum_{m=0}^{I-1} B(\left| g_{n-m+J}(x_o,t) \right|)$$

$$\cos 2\pi\omega(t-(n-m+J)\, \frac{d}{c}\, (x-x_o))\; d\omega dx$$

SECTION V

ANALYSIS

First let us compare the two cases without clipping. For these cases, there is no restriction on M. We observe that $E(G_k)$ is a linear functional of $Q(x)$, the angular distribution of the signal plus noise. As a result we see that

$$E(G_k) - E(G_k^*) = E(G_k^{**}), \text{ where}$$

G_k^{**} is the output when there is only signal, i.e.,

$$Q(x) = 2S\delta(x-x_o), \text{ when the signal is at } x_o$$

Therefore $E(G_k) - E(G_k^*) = I_k^2 S$, since $H_{I_k,J_k}(o,o) = I_k^2$

To obtain $\sigma^2(G_x)$, let $Q(x) = 2S\delta(x-x_o) + Q_N(x)$, when $Q_N(x)$ is the angular distribution of noise alone, then

$$D_{I,J}(x_o,t)=SI^2\rho(t) + \frac{1}{2}\int_{-1}^{1} Q_N(x) \int_o^\infty F(\omega)H_{I,J}\left|\omega(x-x_o),\omega t\right| d\omega dx$$

For large values of I, the noise term in the variance is proportional to I and the noise amplitude in the direction x_o, so that it is in general small compared to the signal term, therefore

$$\sigma^2(G_k) \approx \frac{4S^2 I_k^{\,4}}{T^2} \int_o^T (T-t)\rho^2(t)dt$$

so that

$$R_k \approx \frac{1}{\frac{2}{T}\sqrt{\int_o^T (T-t)\rho^2(t)dt}}$$

i.e., there is no significant difference in performance between the square law and multiplier correlator schemes. If we include the noise term, then R_k is increased by a small term proportional to $1/I_k$, which implies a slight superiority for the square law system.

For small M, e.g., M = 2, the noise term dominates $D_{I,J}$ (i.e., for k = 1, I = 2, for k = 2, I = 1). For k = 1, we have

$$D_{2,o}(x_o,t) = \int_{-1}^{1} Q_N(x) \int_{o}^{\infty} F(\omega)(\cos 2\pi\omega t)\left(1 + \cos 2\pi\omega \frac{d}{c}(x-x_o)\right) d\omega dx$$

For k = 2 we have

$$D_{1,o}(x_o,t) = \frac{1}{2}\int_{-1}^{1} Q_N(x) \int_{o}^{\infty} F(\omega) \cos 2\pi\omega t \, d\omega dx = N\rho(t)$$

$$D_{1,1}(x_o,t) = \frac{1}{2}\int_{-1}^{1} Q_N(x) \int_{o}^{\infty} F(w) \cos 2\pi\omega\left(t + \frac{d}{c}(x-x_o)\right) d\omega dx$$

We can simplify by observing that in general

$$\left| \frac{1}{2}\int_{-1}^{1} Q_N(x) \begin{Bmatrix} \sin \\ \text{or} \\ \cos \end{Bmatrix} 2\pi\omega\frac{d}{c}(x-x_o)\,dx \right| << N \left(= \frac{1}{2}\int_{-1}^{1} Q_N(x)\,dx\right)$$

As a result

$$\sigma^2(G_1) = \frac{16\ N^2}{T^2}\int_{o}^{T} (T-t)\rho^2(t)\,dt$$

$$\sigma^2(G_2) = \frac{2N^2}{T^2}\int_{o}^{T} (T-t)\rho^2(t)\,dt$$

Therefore

$$R_1 = \frac{S}{\frac{N}{T}\sqrt{\int_{o}^{T} (T-t)\rho^2(t)\,dt}}$$

$$R_2 = \frac{S}{\frac{N}{T}\sqrt{2\int_{o}^{T} (T-t)\rho^2(t)\,dt}}$$

In passing, note that for M = 2, if we had used the on target versus off target criterion, R_1 would assume 1/2 of the above

value. Next if we compare $k = 3$ to $k = 4$ for large M, we would get the same result, i.e., there is not significant difference between square law and multiplier-correlator.

Finally let us compare the unclipped and clipped cases for large M, i.e., $k = 1$ and 3 compared or $k = 2$ and 4 compared. Essentially the difference lies in the difference between the expressions for $D_{I,J}$ and $E_{I,J}$. The key difference is the presence of the term $B\left(\left|g_{n-m+J}(x_o,t)\right|\right)$ in the expression for $E_{I,J}$, where the corresponding term in $D_{I,J}$ is unity. Therefore let us summarize the important properties of B. These are $B(o) = \frac{\pi}{4}$, $B(1) = 1$, $B(u)$ is an increasing function of u for $0 \leq u \leq 1$ (B is defined only on the unit interval). Next let us examine $\left|g_n(x_o,t)\right|$. We see that this is the magnitude of a weighted average of complex numbers of unit magnitude. In particular, $\left|g_o(x_o,t)\right| \equiv 1$. If $Q(x)$ is concentrated in a given direction x_o, then $\left|g_n(x_o,0)\right| \approx 1$. If $F(\omega)$ is very narrow band, then $\left|g_n(x_o,t)\right|$ is independent of x_o and t. If $Q(x)$ is constant, then $g_n(x_o,t) = 0$ for $n \neq 0$ (for the narrow band case when $\frac{\omega d}{c} = \frac{1}{2}$). To see what this means, let us represent $E_{I,J}$ by

$$E_{I,J}(x_o,t) = \frac{2}{\pi Z} D_{I,J}(x_o,t) + \frac{4}{\pi^2 Z} \int_{-1}^{1} Q(x) \int_{o}^{\infty} F(\omega) G_{I,J}(x_o,t;x,\omega)\, d\omega dx$$

where

$$G_{I,J}(x_o,t;x,\omega) = \sum_{n=o}^{I-1} \sum_{m=o}^{I-1} \left(B\left(\left|g_{n-m+J}(x_o,t)\right|\right) - \frac{\pi}{4}\right) \cos 2\pi\omega\left(t+(n-m+J)\frac{d}{c}(x-x_o)\right)$$

First let us examine the case when $Q(x)$ is almost uniform. Then

$$G_{I,0} \approx I(1 - \frac{\pi}{4}) \cos 2\pi\omega t$$

and

$$E_{I,0}(x_o,t) \approx \frac{2}{\pi Z} D_{I,0}(x_o,t) + \frac{8I}{\pi^2} (1 - \frac{\pi}{4}) \rho(t)$$

while

$$G_{I,I} \approx 0$$

and

$$E_{I,I}(x_o,t) \approx \frac{2}{\pi^2} D_{I,I}(x_o,t)$$

Therefore

$$R_3 \approx \frac{D_{M,0}(x_o,0) - \frac{S+N}{N} D^*_{M,0}(x_o,0)}{\frac{2}{T}\sqrt{\int_o^T (T-t)\left|D_{M,0}(x_o,t) + M(S+N)\left(\frac{4}{\pi} - 1\right)\rho(t)\right|^2 dt}}$$

$$R_4 \approx \frac{D_{L,L}(x_o,0) - \frac{S+N}{N} D^*_{L,L}(x_o,0)}{\frac{1}{T}\sqrt{2\int_o^T (T-t)\left[\left|D_{L,0}(x_o,t) + M(S+N)\left(\frac{4}{\pi} - 1\right)\rho(t)\right|^2 + D_{L,L}(x_o,t)D_{L,L}(x_o,-t)\right] dt}}$$

Using the approximations to $D_{I,J}$ made previously, and assuming $S << N$, we have

$$R_3 \approx \frac{R_1}{1 + \frac{N}{MS}\left(\frac{4}{\pi} - 1\right)}$$

$$R_4 \approx R_2\sqrt{\frac{2}{1 + \left|1 + \frac{N}{LS}\left(\frac{4}{\pi} - 1\right)\right|^2}}$$

We see then that if $Y_k \equiv \frac{N}{I_k S}\left(\frac{4}{\pi} - 1\right)$ is small compared to unity, clipping has no appreciable effect. However, if it is large then there would be a serious degradation. If the noise is isotropic then Y_k would be small. However, in cases where the medium behaves in such a way that the signal and noise powers in the given direction are greatly reduced relative to other directions (i.e., $Q(\mathbf{x}_o)/I_k S$ is still small), the unclipped systems could still detect the signals, while the performance of the clipped system would be degraded.

To illustrate the effect of clipping, the mean outputs of two systems (without and with clipping) using square law detection, were calculated for a given anisotropic constant frequency background with signals. The array had 200 elements spaced .4 wavelengths apart. As seen from the results (Figure 1), the apparent noise in the high loss direction is considerably increased by the clipping.

SECTION VI

CONCLUSION

1) Square Law Versus Multiplier Correlator

The analysis indicates that from a signal noise ratio point of view, the square law is somewhat better than the multiplier-correlator. However, since the large arrays the difference is negligible, it would be necessary to make an analysis based on the probability distributions of the output to determine if either system has any significant advantage.

2) Use of Clipping

When clipping is used we see that in certain circumstances, e.g., when there is a high medium loss in the signal direction, there could be a serious degradation in detectability. To obtain a more quantitative picture of the situation, the output (square law detector) power distribution, for a particular input power distribution, was calculated for both the unclipped and clipped cases. The results of the example illustrate the clipping loss quite well. (See Figure 1.)

Finally, we can observe that the non-linear effect of clipping on the noise is similar to the effect described by Ogg (21) on the interference on a weak signal by a strong signal when clipping is used.

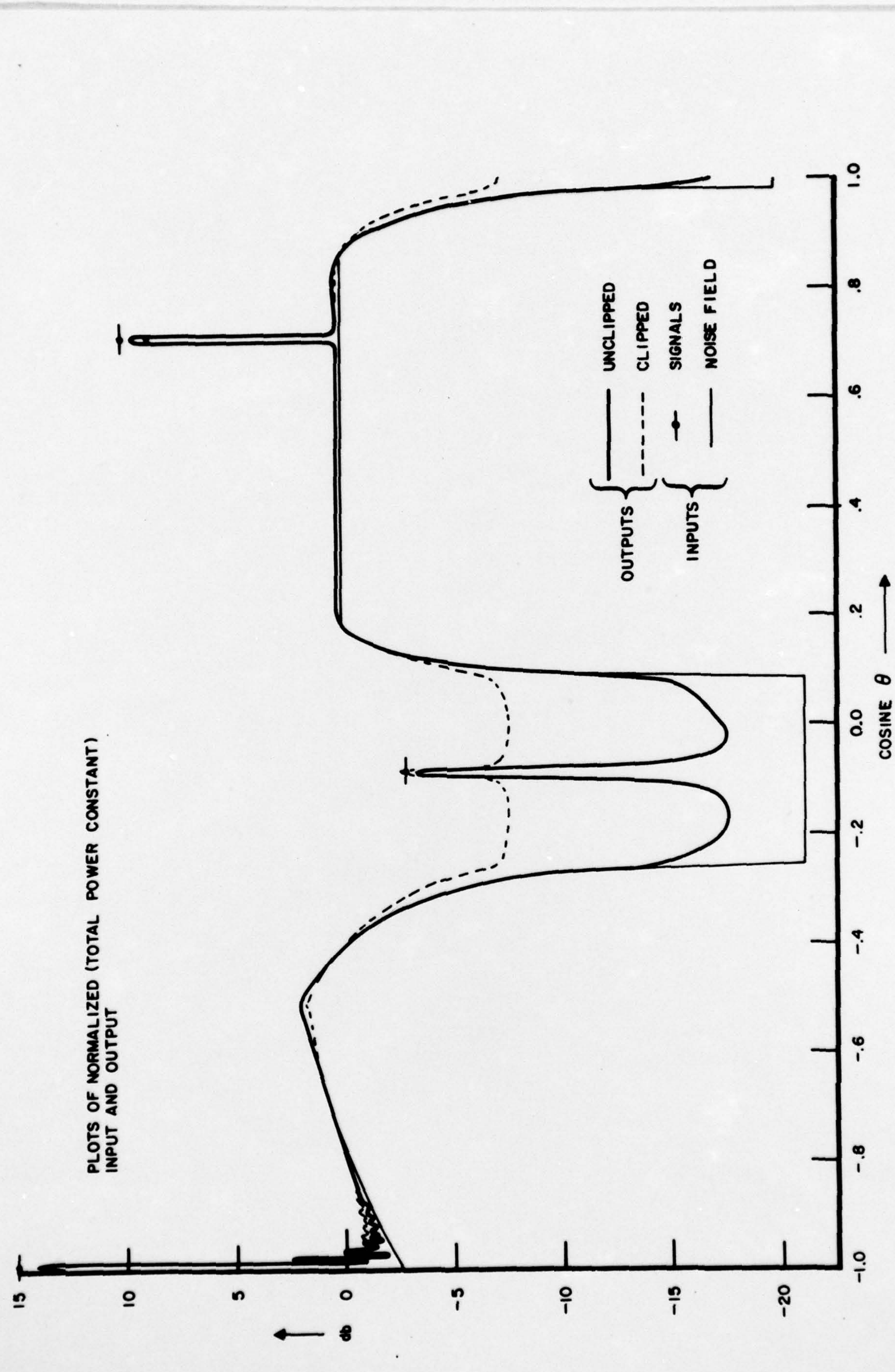


FIGURE 1. EFFECT OF CLIPPING ON S/N IN A NON-ISOTROPIC NOISE FIELD

APPENDIX A

FIGURE OF MERIT

As noted in the introduction, we are using as a figure of merit the deflection criterion of Lawson and Uhlenbeck (22). It must be observed, as Marcum (23) has shown, that this is not a good quantitative criterion for threshold detection. A more accurate procedure would involve first selecting a probability of false alarm, then calculate a threshold setting from the system output distribution with signal absent, and finally calculating the probability that the output will exceed this threshold when the signal is present. In practice, this procedure is extremely difficult to carry out, since the system output distributions are difficult to determine except in certain special cases.

As a result we are forced in practice to use the deflection criterion, at least for a qualitative analysis, i.e., when the signal to noise ratio is small, ≈ 1 or less, then systems with significantly different ratios will perform as predicted by the criterion. Also, if in one system the ratio is $\gg 1$, and in another ≈ 1 or less, then the former can be considered superior. However, if two different systems give large signal-to-noise ratios, then there is no way of comparing then unless something further is known about the underlying distributions.

APPENDIX B

CORRELATION OF CLIPPED GAUSSIAN VARIABLES

Let X_1, X_2 be correlated Gaussian variables of mean zero, and ρ = correlation. We wish to obtain $E(\mathrm{sgn}(X_1,X_2))$.

Lemma 1: $E(\mathrm{sgn}(U)) = 2\ \mathrm{Prob}(U > 0) - 1$, if $\mathrm{Prob}\ (U = 0) = 0$

Proof: Obvious

Without loss of generality, we may assume all deviations equal one.

Theorem: $E(\mathrm{sgn}(X_1X_2)) = \frac{2}{\pi} \sin^{-1}(\rho)$

$$\mathrm{Prob}(X_1X_2 \geq 0) = 2\ \mathrm{Prob}\ (X_1 \geq 0,\ X_2 \geq 0)$$

$$= \frac{1}{\pi\sqrt{1-\rho^2}} \int_0^\infty \int_0^\infty e^{-\frac{x_1^2 + x_2^2 - 2\rho x_1 x_2}{2(1-\rho^2)}} dx_1 dx_2$$

$$= \frac{1}{\pi\sqrt{1-\rho^2}} \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-\frac{r^2(1-2\rho\cos\theta\sin\theta)}{2(1-\rho^2)}} r\, dr d\theta$$

$$= \sqrt{\frac{1-\rho^2}{\pi}} \int_0^{\frac{\pi}{2}} \frac{d\theta}{1-2\rho\cos\theta\sin\theta}$$

$$= \frac{1}{\pi} \cos^{-1}(-\rho)$$

$$= \frac{1}{2} + \frac{\sin^{-1}(\rho)}{\pi}$$

Therefore, by Lemma 1, the theorem is proved.

APPENDIX C

FILTER EFFECT

Let $f(t) = C_o e^{-2\alpha|t|} \cos \alpha\pi\omega_o t$ be the impulse of the band pass filter and let

$$X_j(t) = \prod_{m=1}^{2j+1} \cos 2\omega_m \left(\pi t + (n_1 - n_2)\beta_m\right)$$

We wish to evaluate $Y_j(t_1, t_2)$ given by

$$Y_j(t_1,t_2) = \int_o^{t_1} \int_o^{t_2} f(t_1-\tau_1) f(t_2-\tau_2) X_j(\tau_1-\tau_2) d\tau_1 d\tau_2$$

By a change of variables

$$Y_j(t_1,t_2) = \int_o^{t_1} \int_o^{t_2} f(s_1) f(s_2) X_j(s_2-s_1 + t_1-t_2) ds_1 ds_2$$

Let us ignore the transient effect of the filter, so that

$$Y_j(t_1,t_2) = \int_o^{\infty} \int_o^{\infty} f(s_1) f(s_2) X_j(s_2-s_1 + t_1-t_2) ds_1 ds_2$$

$$= \int_o^{\infty} \int_{-s}^{\infty} f(s) f(s+u) X_j(u+t_1-t_2) du ds$$

$$= \int_{-\infty}^{o} \int_{-u}^{\infty} + \int_o^{\infty} \int_o^{\infty} f(s) f(s+u) X_j(u+t_1-t_2) ds du$$

$$= \int_{-\infty}^{\infty} X_j(u+t_1-t_2) \int_o^{\infty} f(s) f(s+|u|) ds du$$

$$\approx C_1 \int_{-\infty}^{\infty} X_j(u+t_1-t_2) f(u) du$$

To carry out the integration we must first digress slightly.

Let

$$Z_{2n} = \prod_{1}^{2n} \cos A_k$$

$$= \frac{1}{2^{2n}} \prod_{1}^{2n} \left(e^{iA_k} + e^{-iA_k}\right)$$

$$= \frac{1}{2^{2n}} \sum_{m=1}^{2^{2n}} e^{i \sum_k \epsilon_{m,k} A_k}$$

where $\epsilon_{m,k} = \pm 1$, such that the vector $\gamma_m = (\epsilon_{m,1}, ---, \epsilon_{m,2n})$ takes on, as a function of m, all possible values. Therefore

$$Z_{2n} = \frac{1}{2^{2n-1}} \sum_{m=1}^{2^{2n-1}} \cos\left(A_1 + \sum_{2}^{2n} \epsilon_{m,k} A_k\right)$$

(For simplicity we have defined γ_m so that $\epsilon_{m,1} = 1$ for $m = 1, --, 2^{2n-1}$ and -1 for the remaining m).

Now let us examine a typical term in Y_j, using the Z_{2n} representation; such a term W is of the form

$$W = C \int_{-\infty}^{\infty} e^{-2\pi\alpha|u|} \cos(Au+B)\,du$$

$$= 2C \cos B \int_{0}^{\infty} C^{-2\pi\alpha u} \cos Au\,du$$

$$= \frac{4\pi\alpha C \cos B}{(4\pi\alpha)^2 + A^2}$$

Now

$$\cos 2\pi\omega_o u \prod_{m=1}^{2j+1} \cos 2\omega_m\left(\pi u + \pi(t_1 - t_2) + (n_1 - n_2)\beta_m\right)$$

$$= \frac{1}{2^{2j+1}} \sum_{n=1}^{2^{2j+1}} \cos\left[2\pi u\left(\omega_o + \sum_{m=1}^{2j+1} \epsilon_{n,m}\,\omega_m\right)\right.$$

$$\left. + 2 \sum_{m=1}^{2j+1} \epsilon_{n,m}\,\omega_m\left(\pi(t_1 - t_2) \quad (n_1 - n_2)\beta_m\right)\right]$$

Therefore

$$Y_j(t_1,t_2) = \frac{C_1C_o\ 4\pi\alpha}{2^{2j+1}} \cdot \sum_{n=1}^{2^{2j+1}} \frac{\cos\left(2 \sum_{m=1}^{2j+1} \epsilon_{n,m}\omega_m\left(\pi(t_1-t_2) + (n_1-n_2)\beta_m\right)\right)}{(4\pi\alpha)^2 + \left(2\pi\left(\omega_o + \sum_{m=1}^{2j+1} \epsilon_{n,m}\omega_m\right)\right)^2}$$

If we examine the terms of Y_j from the point of view of filtering, observing that $F(\omega_m)$ is narrow band around ω_o, then we see that the only important terms are those were $\Sigma\epsilon_{n,m} = -1$. Furthermore, since the integrand using $Y_j(t_1,t_2)$ is independent of permutations of $\{m\}$, we may simplify as follows:

$$Y_j(t_1,t_2) = \frac{C_1C_o\ 4\pi\alpha}{2^{2j+1}} \binom{2j+1}{j} \cdot \frac{\cos\left(2\left(\sum_{1}^{j} - \sum_{j+1}^{2j+1}\right)\omega_m\left(\pi(t_1-t_2) + (n_1-n_2)\beta_m\right)\right)}{(4\pi\alpha)^2 + \left(2\pi\left(\omega_o + \sum_{1}^{j}\omega_m - \sum_{j+1}^{2j+1}\omega_m\right)\right)^2}$$

If we then normalize, observing that over the frequency band of interest, the filter function is approximately constant, we finally obtain:

$$Y_j(t_1,t_2) = \frac{1}{2^{2j}} \binom{2j+1}{j} \cos\left(2\left(\sum_{1}^{j} - \sum_{j+1}^{2j+1}\right)\omega_m\left(\pi(t_1-t_2) + (n_1-n_2)\beta_m\right)\right).$$

REFERENCES

1. S.O. Rice, "Mathematical Analysis of Random Noise," Bell System Technical Journal, Vol. 23, No. 3, July, 1944 and Vol. 24, No. 1, Jan, 1945.

2. James J. Faran, Jr. and Robert Hills, Jr., "Correlators for Signal Reception," Harvard University, Acoustics Research Laboratory, Technical Memorandum No. 27, Sept. 15, 1952.

3. James J. Faran, Jr., and Robert Hills, Jr., "The Application of Correlation Techniques to Acoustic Receiving Systems," Harvard University, Acoustics Research Laboratory, Technical Memorandum No. 28, Nov. 1, 1952.

4. Melvin J. Jacobson, "Effect of Noise Dependence on Correlation Detection and Measurement," Jl. of the Acoustical Society of America, Vol. 35, No. 3, March 1963.

5. Finn Bryn, "Optimum Signal Processing of Three-Dimensional Arrays Operating on Gaussian Signals and Noise," Jl of the Acoustical Society of America, Vol. 34, No. 3, March 1962.

6. John B. Thomas and Thomas R. Williams, "On the Detection of Signals in Nonstationary Noise by Product Arrays," Jl of the Acoustical Society of America, Vol. 31, No. 4, April 1959.

7. Helen G. Berman and Alan Berman, "Effect of Correlated Phase Fluctuation on Array Performance," Jl. of the Acoustical Society of America, Vol. 34, No. 5, May 1962.

8. David Middleton, "On the Theory of Random Noise, Phenomenological Models I and II, Jl of Applied Physics, Vol. 22, No. 9, September 1951.

9. R. H. Kraichnan, "Pressure Field Within Homogeneous Anisotropic Turbulence," Jl. Acoustical Soc. of America, Vol. 28, No. 1, Jan. 1956.

10. G.M. Corcos, "The Structure of the Turbulent Pressure Field in Boundary-Layer Flows" Jl. of Fluid Mechanics, Vol. 18, Part 3, March 1964.

11. S. Gardner "Surface Pressure Fluctuations Produced by Boundary Layer Turbulence" TRG-142-TN-63-5-R, October 1963.

12. J.L. Stewart and E.C. Westerfield, "A Theory of Active Sonar Detection," Proceedings of the IRE, May 1959.

13. E.C. Westerfield, R.H. Prager, and J.L. Stewart, "Processing Gains Against Reverberation (Clutter) Using Matched Filters," IRE Transactions on Information Theory, June 1960.

14. Pierre Faure, "Theoretical Model of Reverberation Noise," Jl of the Acoustical Society, Volume 36, No. 2, February 1964.

15. D.G. Childers and I.S. Reed, "Optimum Signal Processing in the Presence of Spatial Noise," University of Southern California, Report No. 108, March 1964.

16. Philip Rudnick, "Small Signal Detection in the DIMUS Array," Jl. of the Acoustical Society, Vol. 32, No. 7, July 1960.

17. Willis G. Gore, "Nonlinear Signal Processing in Receiving Arrays," IRE Transactions on Antennas and Propagation, Nov. 1962.

18. R.L. Placket, "A Reduction Formula for Normal Multivariate Integrals," Biometrika 41, 1954.

19. Tze-Chien Sun, "Some Further Results on Central Limit Theorems for Non-Linear Functions of a Normal Stationary Process," Jl. of Math. and Mech., Vol. 14, No. 1, 1965.

20. E. Jahnke and F. Emde, "Tables of Functions with Formulae and Curves," Dover, 1945.

21. Frank C. Ogg, "Steerable Array Radars," IRE Transactions on Military Electronics, April 1961.

22. J.L. Lawson and G.E. Uhlenbeck, "Threshold Signals," McGraw-Hill, 1950.

23. J.I. Marcum, "A Statistical Theory of Target Detection by Pulsed Radar," RAND Research Memo., RM-754, Dec. 1947.